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MODIFIED PROMETHEE V METHOD FOR SUPPLIER PORTFOLIO SELECTION

Frantisek Zapletal
ŠKODA AUTO University, Na Karmeli 1457, Mladá Boleslav, Czech Republic, EU,
frantisek.zapletal@savs.cz
Rijad Trumic
VŠB-Technical University of Ostrava, Faculty of Economics, 17. listopadu 15, Ostrava, Czech Republic, EU,
rijad.trumic.st@vsb.cz
Radim Lenort
ŠKODA AUTO University, Na Karmeli 1457, Mladá Boleslav, Czech Republic, EU,
radim.lenort@savs.cz (corresponding author)

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Abstract: This paper focuses on the problem of supplier portfolio selection where a company has to choose the best possible set of suppliers with respect to various constraints. An intuitive heuristic can suggest to use any of the methods for suppliers ranking and then to put the first one into the portfolio. If some required constraint is not met, then the second supplier according to the ranking is added, and so on, until all the constraints are satisfied. However, such approach can result in a non-optimal decision. The constraints can cause that a combination of the alternatives with lower rankings can be better, than some higher-ranked alternative from the perspective of feasibility. To build the optimization model, the authors of this paper use the PROMETHEE V method: a popular combination of multi-criteria decision making method PROMETHEE and mixed integer programming. However, it is shown that the original PROMETHEE V method, namely the logic under which an objective function is set, is not suitable here and leads to discrimination of suppliers with worse ranking. Therefore, a modification, which brings more reasonable results, is proposed in this paper. A numerical example is used to show the suitability of the proposed approach and compare the results with the original algorithm and also with one prior modification introduced by by other authors in the past. The analysis is further supported by a thorough sensitivity analysis using flexible and parametric programming.

1 Introduction

Suppliers and their quality play a vital role in competitiveness of each production company. Potential troubles with suppliers, like delayed delivery, poor quality of products, difficult communication, or overpriced goods, can be the source of the bottleneck with ominous consequences. Therefore, managers should be very careful to select the most suitable suppliers. In order to do so, many quantitative tools are available to make their selection easier. This problem is a common topic of multi-attribute decision making (MADM). It would be almost impossible to find a MADM method, which has not been used yet to evaluate suppliers. Let us mention at least the methods, which are currently very popular in quantitative support of decision making: Analytic Hierarchy Process (AHP) [1], Analytic Network Process (ANP) [2], TOPSIS [3], ELECTRE [4], PROMETHEE [5]. But, all these studies provide the ranking of suppliers, which is sufficient when the best supplier is identified, or, when companies measure the performance of their current suppliers. However, for many reasons, companies usually do not have only one supplier for all their inputs (either they hedge against risk, or simply because of the availability of the goods). And then, several dependencies and synergistic effects can occur. In this paper, these dependencies and effects will be taken into consideration to identify the best suitable combination of suppliers for a company. The aim is to find the best feasible portfolio of suppliers based on a given set of criteria.

Despite the problem of supplier portfolio selection is by far not as frequent as the ranking problem mentioned above, several studies have also been published. Namely, the authors of [6] have established the model based on the combination of ANP and Data Envelopment Analysis (DEA), and the authors of [7] have presented the model based on genetic algorithms and mathematical programming. Despite these models are very valuable, in our opinion they are very difficult to understand for practitioners, which can limit their use for real-life problems. The model presented in this paper is based on the PROMETHEE method established by [8] and its extension for the portfolio selection presented by [9] (so called PROMETHEE V). The PROMETHEE method is easy to use, since its algorithm is computationally easy and also tractable, see [10], and it provides the ranking of alternatives. Based on this ranking, the portfolio is found using the mixed integer programming (MIP) within the PROMETHEE V method. In this paper, the suitability of the PROMETHEE V method to solve the supplier portfolio selection problem is shown.

As mentioned by [11] and [12], the original PROMETHEE V method suffers from a severe drawback.

Namely, the alternatives with negative values in the PROMETHEE ranking (negative net flows, see Section 2) are discriminated. The authors of [11] have proposed a way how to eliminate this discrimination. However, the proposed method brings undesired biases in favour of large portfolios, see the proof by [13]. The authors of [12] have proposed another solution how to solve the drawback of PROMETHEE V based on the so called c optimal portfolios where the optimal portfolios for a fixed number of selected alternatives c are found. The new model is built on both proposed approaches, i.e. [11] and [12]. The original PROMETHEE V model is transformed according to [1] and explore the c-optimal portfolio. To avoid the biases in favour of large portfolio (like in the original proposal by [11]), the optimisation model is further modified. In the original PROMETHEE V and all its extensions mentioned above, the suitability of a portfolio is determined by the alternatives in this portfolio. To return to the case of supplier portfolio selection, each portfolio of suppliers is evaluated according to the suppliers involved in the portfolio. This is also the reason why the approach by [11] leads to the large portfolio involving all possible suppliers (if some further constraint does not make such solution infeasible), regardless of the supplied quantity. In our opinion, the utility for the supplied company is not generated by suppliers themselves, but through the supplied goods. Therefore, the built model considers that the optimal portfolio is evaluated not only according to the involved suppliers, but the supplied quantity too. In other words, it is supposed that if a supplier delivers 1,000 pieces of some product, or only a single piece, the generated utility is greater in the former case.

This paper brings two main contributions. First, the suitability of the PROMETHEE V method to solve the supplier portfolio selection problem due to its easy and tractable algorithm is demonstrated, and typical segmentation constraints for this problem are identified (the portfolio is constrained by the total budget, demand for products, availability of products at suppliers, size of the portfolio). Second, a new modification of the original PROMETHEE V method [9] is introduced, which is more suitable for the solved problem. On the other hand, it is worth noting that the proposed modification is established for the given application field. Its suitability for other areas must be assessed by a user for each potential application individually.

The paper includes a numerical example, which is solved using the original and modified PROMETHEE V method. Furthermore, a sensitivity analysis of the results for different levels of budget using the flexible and parametric programming is provided.

The rest of the paper is organised as follows. Section 2 recalls the methodology of the PROMETHEE rankings and PROMETHEE V for the portfolio selection. Section 3 presents the PROMETHEE V model for the supplier portfolio selection using both, the original and the new approach. Section 4 provides a numerical example, its results and the sensitivity analysis of the results. The last section, Section 5, concludes the paper and outlines possible directions for the further research.

2 PROMETHEE rankings and PROMETHEE portfolio selection

The family of the PROMETHEE methods belongs to outranking methods of multi-attribute decision making, i.e., its algorithms are based on special (outranking) preference relations. The basic PROMETHEE methods, established by [8], are used to get the rankings of alternatives based on a given (discrete) set of criteria (PROMETHEE I and PROMETHEE II). However, outputs from the PROMETHEE ranking can further be used to solve other decision making problems, like clustering the alternatives [14], efficiency evaluation [15], or portfolio selection [9]. Its variability is not the only advantage of PROMETHEE. It has become very popular mainly due to its very transparent computational procedure that is easy to understand, which is valuable also for practitioners. The popularity of the PROMETHEE method is proved by various fields of real-life applications published so far, see the review paper by [16].

In this section, a brief review of the PROMETHEE algorithms for ranking the alternatives and portfolio selection is provided. More detailed description can be found in [17].

In line with [18], the PROMETHEE ranking can be split into 4 following steps:

Step 1
Preference degrees $P_i(A_t, A_j) = P_i(v_{it} - v_{ij}) \in [0,1]$ are calculated for all pairs of alternatives A with respect to each criterion $i = 1,2, \dots, k$ using preference functions P_i (this function assigns a preference degree to each possible difference in performance values), where v_{it} stands for the performance of the t-th alternative with respect to the i-th criterion. The preference degree says, how much the decision-maker prefers an alternative with better performance in the given criterion to the one with worse performance.

Step 2
The preference degrees are aggregated to preference indices expressing, how much the decision-maker prefers one alternative to another. This is done using the sum product of preference degrees and weights w, see (1) and (2).

Step 3
The preference indices are aggregated to positive and negative flows $(\phi^+ \in [0,1], \phi^- \in [0,1])$ of each alternative, see (1) and (2). The positive flow of an alternative is a mean value of the preference indices comparing this alternative to the others (how much better is the alternative than the others). The other way around, the negative flow of an alternative is a mean value of the

preference indices comparing the remaining alternatives to the one under evaluation (how much worse is the alternative than the others).

Step 4
Due to the fact that the ranking using only the positive and negative flows (PROMETHEE I) provides only a partial ranking (a is preferred to b if $\phi^+(a) \geq \phi^+(b) \wedge \phi^-(a) \leq \phi^-(b)$, where at least one of both inequalities must be strict) these partial flows must be aggregated to the net flows $\phi \in [0,1]$, see (3). The PROMETHEE ranking based on the net flows is called PROMETHEE II and it provides a complete ordering.

The calculations of the described algorithm can be shortly written as follows:

$$\phi^+(A_t) = \frac{\sum_{j=1, j\neq t}^{s} \sum_{i=1}^{k} w_i \cdot P_i(v_{i,t} - v_{i,j})}{s-1} \qquad (1)$$

$$\phi^-(A_t) = \frac{\sum_{j=1, j\neq t}^{s} \sum_{i=1}^{k} w_i \cdot P_i(v_{i,j} - v_{i,t})}{s-1} \qquad (2)$$

$$\phi(A_t) = \phi^+(A_t) - \phi^-(A_t) \qquad (3)$$

where w_i is the weight of the i-th criterion, s is the number of alternatives and k represents the number of criteria.

The authors of [8] have defined the general properties of a preference function. A decision-maker can choose any non-decreasing function P (the greater difference in performances, the greater (or equal) preference strength in favour of the better alternative) with $P(x) = 0$ for $x \leq 0$, with the domain of all real numbers ($x \in \mathbb{R}$) and the range $P(x) \in [0,1]$. In order to make the choice of preference functions simpler for decision-makers, the authors of [8] have proposed some predefined shapes. But, by far the most common shape is the linear one, which allows to consider too small differences in performance values negligible using the indifference threshold q, and, on the contrary, too big differences exceeding the preference threshold p are preferred absolutely and with the same strength, see Figure 1.

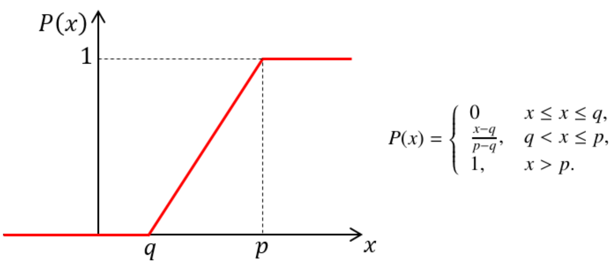

Figure 1 Linear shape of preference function

Multiple alternatives selection using PROMETHEE (PROMETHEE V), introduced by [9], is based on the PROMETHEE II ranking and its net flows ϕ. To find the optimal portfolio of alternatives, mathematical programming must be used. In comparison with the PROMETHEE II algorithm, optimisation models are more complicated to solve. But, the model to solve within the PROMETHEE V is still easily tractable and, in general, more simple than the models proposed by [6], or [7]. Let x_i be a binary decision variable denoting if the i-th alternative is involved in the portfolio, or not. Then, the optimisation model can be written as follows:

$$\begin{aligned} \max \quad & \boldsymbol{\phi}^{\mathrm{T}} \boldsymbol{x} \\ \text{s.t.} \quad & \mathbf{A}\boldsymbol{x} \leq \boldsymbol{b} \\ & \boldsymbol{x} \in \{0,1\}^s \end{aligned} \qquad (4)$$

where the set of constraints with the coefficient matrix $\mathbf{A} \in \mathbb{R}^{n \times s}$ and the right-hand sides $\boldsymbol{b}$ are the segmentation constraints defining the feasibility of a solution. As mentioned in the introduction, dependencies and synergistic effects must be taken into account when selecting multiple alternatives. Each constraint in the model represents a restriction on the portfolio. For example, for a typical asset allocation problem, the constraints can be used to guarantee minimal expected profit and maximal acceptable risk.

3 PROMETHEE model for supplier portfolio selection

In this section, a general optimisation model of the supplier portfolio selection is provided in line with [18]. A company, which needs to deliver m product types by s potential suppliers, is considered. The company is limited by the following constraints:

- total delivery costs cannot exceed the budget b (5d);
- the demand $\boldsymbol{d}$ of the company must be completely satisfied (5b);
- the suppliers have available only limited quantities $\boldsymbol{r}$ of the required product types (5c);
- the portfolio can be restricted in size c (too many suppliers can cause organisational and bureaucratic troubles to the company, on the contrary, too few suppliers increase the risk), (5e).

$$\begin{aligned}
\max \quad & \sum_{j=1}^{s} \phi_j x_j & & \text{(a)} \\
\text{s.t.} \quad & \sum_{j=1}^{s} y_{ij} = d_i & i = 1,2,\dots,m, & \quad \text{(b)} \\
& y_{ij} \leq r_{ij} & i = 1,2,\dots,m, j = 1,2,\dots,s, & \quad \text{(c)} \\
& \sum_{i=1}^{m} \sum_{j=1}^{s} p_{ij} y_{ij} \leq b & & \quad \text{(d)} \\
& \sum_{j=1}^{s} x_j = c & j = 1,2,\dots,s, & \quad \text{(e)} \\
& \sum_{i=1}^{m} y_{ij} \leq M(1-a_j) & j = 1,2,\dots,s, & \quad \text{(f)} \\
& a_j + x_j = 1 & j = 1,2,\dots,s, & \quad \text{(g)} \\
& x_j \in \{0,1\} & j = 1,2,\dots,s, & \quad \text{(h)} \\
& y_{ij} \geq 0 & i = 1,2,\dots,m, j = 1,2,\dots,s, & \quad \text{(i)} \\
& a_j \in \{0,1\} & j = 1,2,\dots,s. & \quad \text{(j)}
\end{aligned} \qquad (5)$$

In the model (5), unlike the general formulation (4), new real variables y_{ij} are used, and they stand for the quantity of the i-th product delivered by the j-th supplier. Thus, the company decides on where to buy the products and how many products are delivered by each supplier. The constraints (5f) and (5g) result from the implication that if there is a supply from a supplier, this supplier is not involved in the portfolio, see Note 1 (M is a sufficiently great prohibitive constant, a_j is a binary dummy variable). The problem (5) is a model of mixed integer programming.

Note 1. The following implication is involved in model (5):

$$\sum_{i=1}^{m} y_{ij} > 0 \Rightarrow x_j = 1, \text{otherwise } x_j = 0,\ j = 1,2,\dots,s. \quad (6)$$

If there is some positive value of y_{ij} for any i, based on the constraint (5f), the corresponding a_j must be equal to zero and, according to (5g), x_j is equal to 1, i.e., the j-th supplier is selected.

The constraints in (5) are relevant for the vast majority of production companies. Each company can also add its individual specific constraints depending on conditions under which it operates. For instance, the following examples of such constraints can be considered:

- each product must be in stock at least at two suppliers for the sake of substitutability;
- total distance to the selected suppliers cannot exceed a given value;
- the shortest route between the selected suppliers cannot exceed a given value;
- delivery costs can also depend on load capacity utilisation (e.g., goods are transported to a customer by trucks and if a truck is not fully loaded, the delivery costs are greater (this also prevents from crumbling the supplied values).

As mentioned in the introduction, two drawbacks of PROMETHEE V make troubles to users.

First, if it is not necessary for the feasibility of a solution, alternatives with negative flows are always excluded from the portfolio because they would decrease the value of the objective function. But, it is not natural to take the zero value of ϕ as a critical threshold if to select the alternative or not. A negative value of ϕ indicates that the negative flow of the given alternative is less than its positive flow (i.e., the weaknesses overweights the strengths), see (3), but it does not necessarily mean, that the alternatives with negative net flows decrease the total utility of the company and vice versa. To face this drawback, the authors of [11] have come with the following modification of the objective function used in PROMETHEE V:

$$\max(\boldsymbol{\phi} + \boldsymbol{q})^{\mathrm{T}}\boldsymbol{x} \quad (7)$$

However, according to [13], the modification using (7) brings the opposite trouble to the original drawback. Namely, if it does not violate any constraint, the optimal portfolio would always include all the alternatives. For the case of (5), it means that each involved supplier would increase the total utility of the company, regardless of the supplied quantity. It can easily happen that it is optimal to deliver `almost zero' quantities from some suppliers, in order to artificially increase the objective function value. This is not desirable. One can admit that, the company can use the constraint (5e) to prevent this problem. However, in our opinion, the method should be applicable even without any additional constraint. Besides that, it is not always easy, or even possible, to set a suitable value for c in (5).

The second drawback is caused by the logic of the objective function as a whole and, in fact, it is the reason why the modification of the objective function proposed by [11] suffers from the troubles mentioned above. The objective function in (4) evaluates a solution according to which suppliers are chosen for the given portfolio. The logic behind is that if a supplier is selected, the evaluation profile of this supplier, including its advantages and disadvantages, is also reflected in 'quality' of its supplies. This idea is reasonable if the decision on a discrete alternative (e.g., a supplier) is not simultaneously accompanied with another decision on some quantitative property (e.g., if a university committee must select, which scientists will be awarded for their research, or municipal elections). However, in the presented supplier portfolio selection (5), the company decides not only on which suppliers are involved in the portfolio, but also on the delivered quantities y_{ij}. From the mathematical point of view, one cannot get the optimal values of variables, which are not included in the objective function. There is also a logical reason, why the function (5a) is not suitable for (5). This will be explained using a simple example. Let me consider the problem described by (5) with $m = 2$ and $s = 3$, i.e., the company requests 2 product types from 3 possible suppliers. Table 1 provides three different feasible solutions of the problem $S1, S2, S3$.

Table 1 Feasible solutions of the numerical example

Solution S_1	Supplier 1	Supplier 2	Supplier 3	Solution S_2	Supplier 1	Supplier 2	Supplier 3	Solution S_3	Supplier 1	Supplier 2	Supplier 3
Net flows:	0.1	0.5	0.9	Net flows:	0.1	0.5	0.9	Net flows:	0.1	0.5	0.9
Product A	20	20		**Product A**	40			**Product A**	10	30	
Product B		40	50	**Product B**		80	10	**Product B**			90

All three solutions bring the same value of the objective function equal to 1.5 using (a) in (5) because all three suppliers are always selected into the portfolio. But, intuitively, $S3$ is the best option and $S2$ is the worst one.

The reason is that, according to the PROMETHEE II ranking, Supplier 3 is the most preferable one (it has the greatest net flow); for example, it provides the best quality of the products and also the best service conditions. Hence, it is reasonable to prefer greater quantities delivered by high-ranked suppliers. Therefore, the authors propose to replace the original objective function (5a) with (8) for the problem described by the model (5).

$$\sum_{j=1}^{s} \phi_j y_{ij} \quad (8)$$

The use of (8) brings also other benefits. First, the modification (7) by [11] does not necessarily favor big portfolios. Second, if the company does not want to explicitly restrict the size of the portfolio, like in (5) using (e), the new optimisation model will not contain any binary variable, the constraints (5f) and (5g) will be excluded, and, thus, the problem will be a linear programming problem, which will be simpler, smaller and faster to solve. But, in this paper, all the constraints from (5) will be kept to provide the sensitivity analysis of the results for changing portfolio size:

$$\max \sum_{j=1}^{s} (\phi_j + q)\, y_{ij} \quad (9)$$
$$\text{s.t.} \quad (5b) - (5j).$$

where q is set in line with (7).

The objective function of the model (9) assigns 77, 53, 97 to three solutions in Fig. 2 $S1, S2, S3$, respectively. This result confirms the intuitive reflection above.

At the end of this section, it is worth emphasizing that the proposed modification is suitable for the considered decision making problem. But it cannot be automatically used for other problems without further analysis. As mentioned above, for some problems, the original PROMETHEE V model is more suitable. As well as it is possible that one can face a problem, for which none of the presented models makes sense, and which will require some completely unique approach.

4 Numerical example

In this section, a numerical example of the supplier portfolio selection is provided. Namely, the example presented by [19] is used. The original authors have used this example to demonstrate their DEA-based method to evaluate the suppliers. The original data to get the PROMETHEE rankings are used, and extended with necessary input data for the algorithms and models presented in Sections 2 and 3 similarly to [18], where the example has been solved using the original PROMETHEE V method.

The modeled company evaluates 18 potential suppliers S1-S18 using 5 quantitative criteria:

- Supply variety [number of provided product types] (maximizing);
- Quality [% of non-defect products] (maximizing);
- Distance [km] (minimizing);
- Delivery [% of products delivered in time] (maximizing);
- Price index [%] (minimizing).

The same importance is considered for all the criteria, i.e., each of them has a weight w equal to 0.2. All the criteria are treated using the linear shape of the preference function, see Figure 1. The performances of the suppliers in the given criteria, together with the thresholds for the preference functions, are displayed in Table 2.

Table 2 Performances of the suppliers and the threshold values p and q

	C1	C2	C3	C4	C5		C1	C2	C3	C4	C5
q	5	0	30	0	0	w	0.2	0.2	0.2	0.2	0.2
p	20	5	1000	20	11						
S1	2	100	249	90	100	S10	3	97.5	588	100	100
S2	13	99.8	643	80	100	S11	10	100	241	95	100
S3	3	100	714	90	100	S12	7	99.9	567	98	100
S4	3	100	1809	90	100	S13	19	100	567	90	100
S5	24	99.8	238	90	100	S14	12	91.9	967	90	100
S6	28	96.6	241	90	100	S15	33	100	635	95	80
S7	1	100	1404	85	100	S16	2	100	795	95	80
S8	24	100	984	97	100	S17	34	100	689	95	80
S9	11	99.9	641	90	100	S18	9	99.4	913	85	100

Using (1), (2), and (3), the PROMETHEE II ranking of the suppliers is calculated, see Table 3. In line with (7), the

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transformation q is set to $\left|\min_j\{\phi_j\}\right| + 0.0001 = 0.3259$, without loss of generality. The final adjusted values of the objective function coefficients for (9) are provided in Table 3.

Table 3 The results of the PROMETHEE II analysis

Rank	Supplier	ϕ_j	$\phi_j + q$	Rank	Supplier	ϕ_j	$\phi_j + q$
1	S15	0.4008	0.7267	10	S1	-0.0167	0.3092
2	S17	0.3953	0.7212	11	S9	-0.0438	0.2821
3	S10	0.2204	0.5463	12	S16	-0.0671	0.2588
4	S5	0.1546	0.4805	13	S3	-0.0994	0.2265
5	S8	0.0915	0.4174	14	S2	-0.1399	0.186
6	S11	0.0794	0.4053	15	S18	-0.1866	0.1393
7	S6	0.0513	0.3772	16	S4	-0.2879	0.038
8	S13	0.0498	0.3757	17	S7	-0.3005	0.0254
9	S12	0.0246	0.3505	18	S14	-0.3258	0.0001

The company needs 10 products P1-P10 for production in quantities d_i provided in Table 4. Each product can be delivered by at least two suppliers to avoid trivial results. The selling prices per one product can differ with suppliers, see the values typeset with upper indices in Table 4. The last remaining input value for (9) is the upper bound for the total delivery costs b. It is assumed that the company is not able to set this value. Therefore, in the first instance, the model (9) is run without the budget constraint (5d) and the obtained results are further used for the sensitivity analysis exploring the restrictive effect of the budget constraint.

Table 4 Available numbers of products with their selling prices (in bold)

	S1	S2	S3	S4	S5	S6	S7	S8	S9	d_i
P1	10 **6**	0	0	20 **8**	0	0	0	0	0	15
P2	0	0	30 **5**	40 **6**	0	0	0	30 **7**	0	60
P3	0	0	0	50 **7**	0	0	20 **6**	0	0	70
P4	0	20 **2**	0	0	100 **3**	0	0	0	0	120
P5	0	0	0	0	40 **5**	30 **5**	0	0	40 **4**	80
P6	0	25 **6**	0	0	50 **5**	0	0	60 **5**	0	70
P7	0	0	0	0	0	0	0	50 **9**	0	40
P8	0	0	0	0	0	0	90 **4**	0	0	100
P9	0	0	0	0	0	0	0	0	0	100
P10	0	0	0	0	0	0	0	0	0	100
	S10	S11	S12	S13	S14	S15	S16	S17	S18	
P1	0	0	0	30 **7**	0	0	0	0	0	
P2	0	80 **4**	0	0	60 **6**	0	0	0	0	
P3	0	0	0	50 **9**	0	0	0	0	0	
P4	0	50 **1**	0	0	80 **2**	0	0	100 **3**	0	
P5	0	0	0	50 **5**	0	50	0	30 **4**	0	
P6	60 **4**	0	0	0	0	0	60 **3**	0	0	
P7	60 **10**	0	50 **8**	0	0	50 **9**	0	0	0	
P8	80 **5**	0	0	0	0	0	0	90 **5**	0	
P9	120 **4**	0	0	0	0	80 **5**	0	0	70 **3**	
P10	80 **4**	0	0	0	0	0	0	0	100 **3**	

In line with [12], the model is run for different values of portfolio size c, in order to get the so called c-optimal portfolios. In the first instance, the model is run without the constraint (5e) to get the upper bound $\bar{c}$ for c. Then the model is run repeatedly with gradually decreasing c until the feasible solution exists (i.e., for $c = \bar{c}, \bar{c} - 1, \bar{c} - 2, \dots, \underline{c}$, where $\underline{c}$ is the minimal c, for which the model remains feasible).

The model (9) is solved using the MIP solver of GAMS software and a computer with I7 Intel processor 2.59GHz, 16GB RAM and Windows 10 x64 OS. The model contains 216 variables (180 real variables and 36 binary variables) and 228 constraints (excluding non-negativity constraints and binary constraints) in total.

4.1 Results and discussion

The results of the model (9) without the budget constraint for different values of c are provided in Table 5. It can be seen that the optimal portfolio without the constraint on size contains 9 suppliers (thus, $\bar{c} = 9$) and the model is feasible for $c \geq 7$ (thus, $\underline{c} = 7$). The corresponding optimal values of the objective functions are shown in Table 6.

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One can expect that when c is decreased by 1, one (the least suitable) supplier will be excluded from the portfolio and the remaining suppliers will still be selected. But, the results in Table 5 shows that this is not true in general, see the supplier S11 that is in the c-optimal portfolios for $c = 7{,}9$, but not for $c = 8$.

Table 5 The optimal portfolios based on the new proposed approach and their comparison with the previous approaches by [9] and [11] without bound c (i.e., $c \leq 18$), and for $c = 7$ and $c = 8$

	The approach proposed in this paper																	
	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18
$c = 7$	0	0	0	0	1	0	1	0	0	1	1	0	1	0	1	0	0	1
$c = 8$	0	0	0	1	1	0	0	1	0	1	0	0	1	0	1	0	1	1
$c \leq 18$	0	0	0	1	1	0	0	1	0	1	1	0	1	0	1	0	1	1
	The original PROMETHEE V by [9]																	
	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18
$c = 7$	0	0	0	0	1	0	1	0	0	1	1	0	1	0	1	0	0	1
$c = 8$	0	0	0	1	1	0	0	1	0	1	0	0	1	0	1	0	1	1
$c \leq 18$	0	0	0	1	1	**1**	0	1	0	1	1	**1**	1	0	1	0	1	1
	The approach proposed by [11]																	
	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18
$c = 7$	0	0	0	0	1	0	1	0	0	1	1	0	1	0	1	0	0	1
$c = 8$	0	0	0	**0**	1	0	**1**	**0**	0	1	**1**	0	1	0	1	0	1	1
$c \leq 18$	**1**	**1**	**1**	1	1	**1**	**1**	1	**1**	1	1	**1**	1	**1**	1	**1**	1	1

Table 6 The total utility expressed by the objective function in (9)

	This study	Brans and Mareschal [9]	Mavrotas et al. [11]
$c = 7$	382.57	170.79	170.79
$c = 8$	431.12	171.55	171.55
$c \leq 18$	442.14	173.82	179.11

Table 5 includes also the results of the problem solved using the original PROMETHEE V method and the modified version by [11] (differences are typeset in bold). The most differences can be found in the models unconstrained in size. Unlike the approach proposed in this paper, the results of the model solved with the original approach [9] include 11 suppliers in the optimal portfolio (S7 is replaced by S6 and, in addition, S8 and S12 are added). The results using the approach by [11] bring no surprising results - all the suppliers under consideration are selected to the optimal portfolio. Two reversals occur for $c = 8$: S4 and S8 are replaced by S7 and S11 for the model based on [11], meanwhile the results based on [9] are identical to the new proposed approach in this case. The optimal portfolios are the same for the smallest feasible size, i.e., $c = 7$. The optimal values of the objective functions can be found in Table 6. For the sake of comparability, all the optima are calculated using the objective function in (9), i.e., the values obtained by two compared approaches must be recalculated accordingly. It is not surprising that the new approach brings the best values. But, it is worth noticing that the new approach performs more than twice better in comparison with the other two models for all three scenarios of c (note that this is approximately the same difference as in the example in Figure 2 where the solutions could be evaluated intuitively).

Despite the optimal portfolios for all three compared approaches are identical for $c = 7$, the structure of supplies y_{ij} differs for the new approach (M1) and others (M2,M3), as it is signaled by the values in Table 6, see Table 7. Differences in values are typeset in bold. One difference deserves a special comment: based on the input data, S18 must be selected because another supplier who provides P10, i.e, S10, cannot satisfy the whole demand equal to 100. However, this supplier performs very poorly according to the PROMETHEE ranking (see Table 3). In spite of this, the approaches designated as M2 and M3 assign the whole demand to S18 because the delivered quantity does not influence the value of the objective function. The solution M1 should be definitely preferred. It is worth noting that this solution is an alternative optimal solution even for M2 and M3, but the probability that this solution is found by their corresponding models tends to zero.

Table 7 The optimal distribution of supplies for c=7 and three compared approaches

		S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18
P1	M1	0	0	0	0	0	0	0	0	0	0	0	15	0	0	0	0	0	0
	M2,M3	0	0	0	0	0	0	0	0	0	0	0	**0**	**15**	0	0	0	0	0
P2	M1	0	0	0	0	0	0	0	0	0	0	60	0	0	0	0	0	0	0
	M2,M3	0	0	0	0	0	0	0	0	0	0	60	0	0	0	0	0	0	0
P3	M1	0	0	0	0	0	0	20	0	0	0	0	0	50	0	0	0	0	0
	M2,M3	0	0	0	0	0	0	20	0	0	0	0	0	50	0	0	0	0	0
P4	M1	0	0	0	0	100	0	0	0	0	0	20	0	0	0	0	0	0	0
	M2,M3	0	0	0	0	**70**	0	0	0	0	0	**50**	0	0	0	0	0	0	0
P5	M1	0	0	0	0	30	0	0	0	0	0	0	0	0	0	50	0	0	0
	M2,M3	0	0	0	0	**40**	0	0	0	0	0	0	0	0	0	**40**	0	0	0
P6	M1	0	0	0	0	10	0	0	0	60	0	0	0	0	0	0	0	0	0
	M2,M3	0	0	0	0	10	0	0	0	**0**	**60**	0	0	0	0	0	0	0	0
P7	M1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	40	0	0	0
	M2,M3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	40	0	0	0
P8	M1	0	0	0	0	0	0	20	0	0	80	0	0	0	0	0	0	0	0
	M2,M3	0	0	0	0	0	0	**90**	0	0	**10**	0	0	0	0	0	0	0	0
P9	M1	0	0	0	0	0	0	0	0	0	20	0	0	0	0	80	0	0	0
	M2,M3	0	0	0	0	0	0	0	0	0	**10**	0	0	**0**	0	0	0	0	0
P10	M1	0	0	0	0	0	0	0	0	0	80	0	0	0	0	0	0	0	20
	M2,M3	0	0	0	0	0	0	0	0	0	**0**	0	0	0	0	0	0	0	**100**

Another analysis of the optimal supplies' structure is done for three scenarios of c values (again for $c = 7{,}8$ and $c \leq 18$). Figure 2 shows the results of this analysis. If the size of the portfolio is reduced from 9 to 8, only one change occurs – S8 is excluded from the portfolio and its quantity is covered by S11. On the contrary, when c is reduced to 7, the exclusion of S17 leads to many complex changes in the optimal portfolio.

As to the total delivery cost, if there is no budget constraint b set upon the model, the total costs for $c = 7, 8, 9$ are equal to 3,625; 3,825; 3,765 respectively. The minimum of the delivery cost, for which model (9) remains feasible, equals 2905, i.e., (9) brings more expensive solutions almost by 25%. The company can reduce these costs using a budget bound b (5d). An effect of this bound on the optimal solution of the model (9) is explored within the sensitivity analysis in Section 4.2.

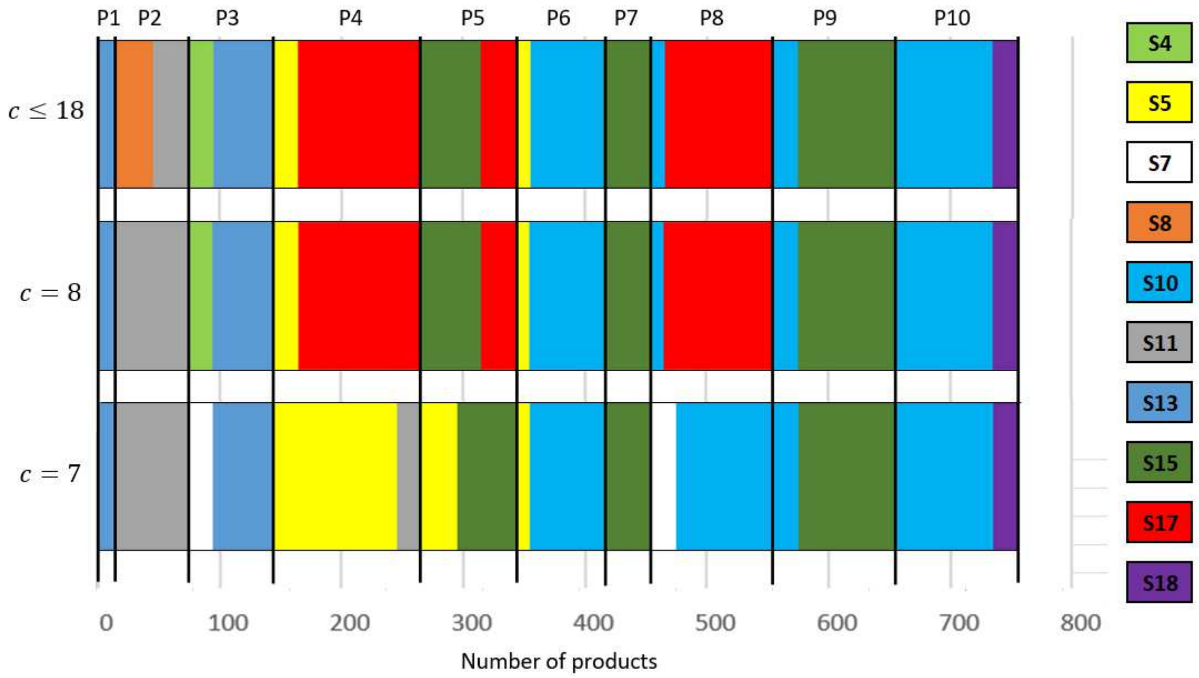

Figure 2 Optimal structure of supplies for the numerical example solved by the new proposed approach with different sizes of portfolio

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4.2 Sensitivity analysis

In this section, it is explored how the optimal solution of the solved numerical example changes when the company limits its total delivery cost by various values. It has been shown that if the company does not restrict the budget, nor the portfolio size, the highest cost equals 3,765 units. On the contrary, the minimum value of the cost, when ignoring the PROMETHEE rankings of the suppliers, is equal to 2,905. It is reasonable to expect that the company requires to reach the budget *substantially lower* than the one given by model (10). In this way, it is possible to avoid determining a precise value. In line with [20], such vague constraint can be solved using the flexible programming approach where the uncertain relation is expressed by a fuzzy set. Namely, the basic Verdegay's model with vague constraints is used here. *Substantially lower than 3,765* is replaced by the non-increasing fuzzy interval depicted in Figure 3. The membership degree α represents to what extent the company is satisfied with the value of costs. Certainly, it is absolutely satisfied when the cost equals the absolute minimum of 2,905 units and it is not satisfied at all if the cost is greater than or equal to 3,765 units.

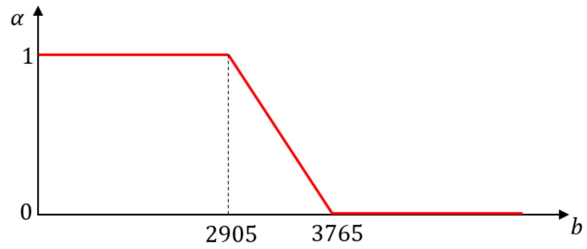

Figure 3 Fuzzy interval describing the uncertain 'Substantially lower than 3,765' relation

According to [20], the deterministic form of the given vague constraint is as follows:

$$\sum_{i=1}^{m}\sum_{j=1}^{s} p_{ij}y_{ij} \leq 3765 - 860\alpha \qquad (10)$$

where 860 is the difference between the cost of absolute dissatisfaction (3,765) and absolute satisfaction (2,905).

Table 8 The results of the numerical example with flexible constraint on the total delivery cost (OF = objective function value)

α	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	OF
0	0	0	0	1	1	0	0	1	0	1	1	0	1	0	1	0	1	1	442.14
0.1	0	0	0	1	1	0	0	1	0	1	1	0	1	0	1	0	1	1	441.79
0.2	1	0	0	0	1	0	1	0	0	1	1	0	1	0	1	1	1	1	428.02
0.3	1	0	0	1	0	0	1	0	0	1	1	0	1	0	1	1	1	1	424.62
0.4	1	0	0	1	0	0	1	0	0	1	1	0	1	0	1	1	1	1	410.05
0.5	1	0	0	1	0	0	1	0	0	1	1	0	1	0	1	1	1	1	393.47
0.6	1	0	0	1	0	0	1	0	0	1	1	1	1	0	1	1	1	1	364.48
0.7	1	0	0	1	0	0	1	0	0	1	1	1	1	0	1	1	1	1	329.47
0.8	1	0	0	1	0	0	1	0	1	1	1	1	0	1	1	1	1	1	293.42
0.9	1	1	0	1	0	0	1	0	1	1	1	1	1	0	1	1	1	1	241.56
1	1	1	0	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	180.45

The model (9) will be solved again, but with (d) constraint replaced by (10) for $\alpha \in [0,1]$ (particularly for $\alpha = 0, 0.1, 0.2, \ldots, 1$). The results are provided in Table 8. It is not surprising that the optimal values of the objective function decrease with increasing level of satisfaction with the budget constraint (10) since the model is more restricted. On the other hand, the portfolio size increases with increasing α. But, it is worth noting that the optimum of the objective function for maximum satisfaction with the budget constraint (i.e., $\alpha = 1$) is still greater than the values of the same function for the optimal solutions of the compared approaches without the budget constraint in Section 4.1, see Table 6.

5 Conclusions

In this paper, the authors focused on the supplier portfolio selection problem. Namely, it was shown how the PROMETHEE outranking method can be used to solve this kind of economic problem. A general model with the constraints applicable to the vast majority of production companies has been built, and, using this model, it was demonstrated that the original PROMETHEE V method [9] can distort the final decision. This drawback stems from the logic how the utility of portfolios is measured. In the original method, the evaluation is based only on the selected suppliers regardless of the quantities delivered by these suppliers. This fact strongly favors large portfolios by crumbling the supplies among the suppliers with positive values of the net flows resulting from the PROMETHEE II ranking. The second drawback, which has already been many times discussed by researchers, is that the original PROMETHEE V discriminates the suppliers with negative values of the net flows. Therefore, the authors came with the modification of the objective function of the PROMETHEE V optimisation model. To deal with the former drawback, each portfolio was evaluated using not only its structure, but it was also taken into account how many items the selected suppliers supply. The latter drawback was eliminated using the approach by [11]. Despite the authors of [13] has proved this approach unsuitable because it favors large portfolios even more than the original method, it is convenient for the proposed approach when it is applied together with the proposed change in the logic of portfolio evaluation.

Using the numerical example, the proposed approach has been compared with the original one. The optimal decisions of the compared approaches can differ in the portfolio structure and supplied quantities. As to the optimal structure of suppliers, the new approach brings more satisfiable results if the portfolio size is not explicitly restricted by the constraint (not all the suppliers with the positive values of the PROMETHEE net flows are necessarily selected, unlike the original approach). Concerning the optimal supplied quantities, the results of the new and original approaches differ a lot because the original model does not involve these quantities in the objective function. In our opinion, the new approach brings the results closer to practice for the considered economic problem.

The sensitivity analysis, which was done using the flexible fuzzy constraint on the budget, showed that the size of the portfolio increases with a degree of satisfaction with the budget level. Moreover, the total utility of the optimal portfolios of suppliers was greater than the total utility brought by the original PROMETHEE V recalculated using the new proposed objective function for all degrees of satisfaction with the budget level.

It was shown that the PROMETHEE V approach is a suitable method to solve the supplier portfolio selection problem with the proposed modification of an objective function, which takes into account not only the structure of portfolio, but also the quantities delivered. The solution is non-trivial and it brings a consistent and desired quantitative support for the decision. On the other hand, if only the portfolio structure matters, and no other accompanying decisions, e.g., on quantities related to the alternatives, must be done, the new approach is useless, and the original logic must be used for the objective function. The authors are also aware of another potential weakness of the proposed approach. It was supposed that the utility of each portfolio depends on the PROMETHEE II rankings and delivered quantities. But, one can admit that, sometimes, the supplied quantities are not the main driving factor. For example, if a construction company plans to order 1,000 nails and only one automatic nailer, it can be reasonable to suppose that the marginal utility brought by delivery of a single item differs in favor of substantially more expensive and reusable nailer. The future research can be focused on other possible modifications of the objective function used in the PROMETHEE V method for various real-life economic applications. Another direction of extension could be the extension of the model with the significance (weights) of the suppliers as the authors of [21] proposed.

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